Exercise 29

Differentiate f and find the domain of f.

$$f(x) = \ln(x^2 - 2x)$$

Solution

Recognize that only the logarithm of a positive number can be taken.

$$x^2 - 2x > 0$$
$$x(x - 2) > 0$$

The critical points are then 0 and 2. Partition the number line at these numbers and then test whether the inequality is true within each of the intervals.



Therefore, the domain of the function is

$$(-\infty,0) \cup (0,\infty).$$

Take the derivative of the function with respect to x by using the chain rule.

$$f'(x) = \frac{d}{dx} \left[\ln(x^2 - 2x) \right]$$

= $\frac{1}{x^2 - 2x} \cdot \frac{d}{dx} (x^2 - 2x)$
= $\frac{1}{x^2 - 2x} \cdot (2x - 2)$
= $\frac{1}{x(x - 2)} \cdot 2(x - 1)$
= $\frac{2(x - 1)}{x(x - 2)}$