## Exercise 29

Differentiate $f$ and find the domain of $f$.

$$
f(x)=\ln \left(x^{2}-2 x\right)
$$

## Solution

Recognize that only the logarithm of a positive number can be taken.

$$
\begin{aligned}
& x^{2}-2 x>0 \\
& x(x-2)>0
\end{aligned}
$$

The critical points are then 0 and 2. Partition the number line at these numbers and then test whether the inequality is true within each of the intervals.


Therefore, the domain of the function is

$$
(-\infty, 0) \cup(0, \infty)
$$

Take the derivative of the function with respect to $x$ by using the chain rule.

$$
\begin{aligned}
f^{\prime}(x) & =\frac{d}{d x}\left[\ln \left(x^{2}-2 x\right)\right] \\
& =\frac{1}{x^{2}-2 x} \cdot \frac{d}{d x}\left(x^{2}-2 x\right) \\
& =\frac{1}{x^{2}-2 x} \cdot(2 x-2) \\
& =\frac{1}{x(x-2)} \cdot 2(x-1) \\
& =\frac{2(x-1)}{x(x-2)}
\end{aligned}
$$

